Benchmark: Bouncing Ball

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Abstract

The bouncing ball is an academic benchmark frequently used for illustration and testing of algorithms. Despite being extremely simple, it exhibits a several key phenomena that are typical for hybrid systems and reachability.

Category: academic **Difficulty:** low[1]

1 Context and Origins

(application domain, existing real-world systems, references)

The bouncing ball is widely used in teaching and presentations. The description is an idealized, purely Newtonian model of a point mass with collision. Other effects such as friction, compression, sticking etc. are neglected. We are not aware of any particular citations or definitive origin.

2 Brief description

(controller and plant overview, properties to check, example trajectories)

The system consists of a ball (point mass) that is initially at rest at some height above the ground. As gravity acts on the ball, it accelerates towards the ground. The collision with the ground results in a reversal of the sign of its velocity, and the loss of energy during the collision is approximated by scaling the velocity by a constant factor $c \leq 1$.

Reachability properties to be checked:

1. boundedness of the reachable set: Approximating the reachable set with low accuracy can lead do a rapidly diverging, unbounded set. The difficulty increases as $c \to 1$.
2. convergence: Verify that the velocity decreases from one jump to the next. Similar for the max. height between two consecutive jumps.

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†For the version number, use the number before the decimal point to indicate major revisions, and the one after it for minor revisions. Please also include a date!

[1] Indicate as appropriate: category (academic, industrial), difficulty (low, medium, high, challenge)

3 Key observations

(characteristic phenomena exhibited by the benchmark, particular difficulties)

The bouncing ball shows Zeno behavior, i.e., the number of jumps in a fixed interval of time eventually becomes unbounded for $c < 1$. This can be made explicit in the model by adding a clock.

Several properties can serve as quality indicators for reachability algorithms and tools:

1. accuracy: Since analytic solutions are available, the accuracy of the reach set approximation can be evaluated. For bounded model checking this can be the max. height after the nth jump.

2. termination of reachability algorithms: Check whether a cover of the reachable set can be computed over an infinite time horizon. The number of jumps of the ball until termination combined with the accuracy of the final result are indicators for the convergence/accuracy trade-off of the algorithm.

3. Zenoness: Adding a clock to the system, check whether time can diverge.

4 Outlook

(future developments, possible extensions)

It is possible to add nondeterministic disturbances to both the continuous and discrete dynamics.

In a future revision, the authors intend to add monitor automata for showing the different properties given above.

A Appendix

(Detailed description, equations, link to model file)

The model has two real-valued state variables: the position above ground x and the velocity v. The dynamics of the system are

$$\dot{x} = v, \qquad (1)$$

$$\dot{v} = -g, \qquad (2)$$

where $g \geq 0$ is the real-valued gravitational constant. It holds at all times that $x \geq 0$. A collision with the ground occurs when $x = 0 \land v < 0$. The collision changes the velocity according to

$$v := -cv, \qquad (3)$$

where c is a real-valued factor that models the loss of energy from the collision, with $0 \leq c \leq 1$. Typical values for the coefficients are $g = 1, c = 3/4$.

A hybrid automaton model in SX format (suitable for the tool SpaceEx) is available at `http://cps-vo.org/group/ARCH/`[2].

[2]Additional files can be added during the submission in EasyChair, and be posted later on the ARCH website. Please include a readme.txt that describes the content and purpose of the individual files.